On the Definition of Motion Parallax

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In recent years experimental interest in motion parallax has increased, following the rediscovery of the idea to yoke stimulus motion to head movement. Moreover, definitions of motion parallax are included in most introductory textbooks of perception, but they are not all in accordance. We compare the definitions with contemporary research, which indicates how depth and motion perception are dependent on the conditions of stimulation.

1. Introduction

Our talk (Jan. 27, 2004) given at the meeting of the Vision Society of Japan was based on our two recent papers. One is by Ono and Wade (submitted)¹⁾, and another is by Ono and Ujike (submitted)²⁾.

Ono and Wade (submitted)¹⁾ reviewed historical discussions of motion parallax, and related them to modern treatments of it in textbooks. The possibility of head movement leading to depth perception was clearly stated in the late seventeenth century, and descriptions of the apparent motion that occurs with head movement were noted in the early nineteenth century. That is, the term motion parallax was used with respect to the depth and motion seen from self-generated retinal image motion.

Rohault $(1671)^{3)}$ described how distance could be determined from the changes in direction consequent upon the movement of a single eye. An even more precise account was given by La Hire $(1694)^{4)}$, who used the term 'parallax' to indicate how near objects can be distinguished in depth. The direction of

displacement of objects nearer than and farther from the fixated object was clearly specified, and the link between the direction of displacement and the relative depth was duly noted. Herschel $(1833)^{5}$ described the consequences of viewing a scene through a moving window — one that is used extensively in contemporary illustrations. Not only did he specify the directions of (what he called) parallactic motion but he also implied that some allowance for this could take place, so that the motion perceived could be attributed to objects or to the observer. Herschel's statements indicate that he was more aware of the subtleties of motion parallax than are many modern writers. The motion with respect to the observer is allocated to depth, motion, or both. It was shortly after Herschel's account that Wheatstone (1838)⁶⁾ addressed the problem of the perception of depth by those who did not have recourse to binocular vision. His solution was that successive projections associated with head movements "may assist in suggesting to the mind the distance of the object" (Wheatstone, 1838, p. 377)⁶⁾.

With this historical legacy, it is surprising that

wide divergences in the definition of motion parallax are found in present day introductory textbooks on perception. Ono and Ujike (submitted)²⁾ provided an empirical basis to evaluate the definitions of motion parallax. Using their findings as the focus of our discussion, we suggest a more representative definition of motion parallax at the end of this paper.

2. Early experimental studies of motion parallax

Bourdon (1898, 1902)^{7,8)} demonstrated that judgments of the separation between two spots of light at different distances and equated in visual angle were made accurately when the head moved, but not when the head remained stationary. A few years later, Heine $(1905)^{9}$ controlled the rate of retinal image motion with respect to head movement (see Fig. 1). He mechanically yoked lateral body movement to stimulus movements using a shoulder harness. The closer stimulus physically moved in the same direction and the far one physically moved in the opposite direction to head/body movement. That is, the normal relationship between retinal image displacement and head movement was reversed. Under these conditions, the perception of depth was opposite to the physical depth; the near stimulus appeared farther away than the (physically) far one, which provided convincing evidence that the retinal image motion produced by a head/body movement is a cue to depth perception.

Even fifty years after Bourdon (1898)⁷, the complete control of head movement (and therefore retinal image motion) proved difficult. Graham, Baker, Hecht, and Lloyd (1948)¹⁰ and Zegers (1948)¹¹ measured motion thresholds for motion parallax thresholds another way: "For experimental purposes, it is convenient to use a





B. Front View





situation in which the S remains stationary while the objects move" (Graham et al., p. 207)¹⁰⁾. A comment is offered later regarding the assumption that the relative motion threshold with the head stationary is a good estimate of parallactic depth threshold. Ittelson (1960)¹²⁾ provided a more detailed geometrical analysis of motion parallax. He remarked that the consequences of head movement could be the perception of relative depth or apparent motion.

As indicated above, early visual scientists were aware of the usefulness of self-generated retinal image motion as a cue to depth and there was a mild but continued interest, which has been recently revived. One reason for the recent upsurge of research is Rogers and Graham's $(1979)^{13}$ development of a technique similar to that devised by Heine (1905)⁹⁾. It consists of electronically yoking dot movements on a screen to lateral head movements, and the perception produced is one of surfaces with apparent depth analogous to that produced by the random dot stereograms devised by Julesz (1971)¹⁴⁾. A comprehensive review of research since Rogers and Graham's article can be found in Howard and Rogers $(2002)^{15}$, and is not repeated here. Instead, we focus on descriptions of motion parallax that are given in contemporary introductory textbooks on perception. As we show shortly, the textbooks we surveyed mention motion parallax, but there are wide divergences in the manner in which it is defined.

3. Introductory textbook accounts of motion parallax

Ono and Wade (submitted)¹⁾ discussed the illustrations and definitions of motion parallax found in nine current introductory textbooks on perception (see their Figures 7 and 8). Below are four samples of the definitions they discussed, which show how varied they are. For a more complete list with illustrations, see Ono and Wade¹⁾.

"The apparent relative motion of objects in the visual field as the observer moves his head or body." (Coren, Ward and Enn, 1999, p. 569)¹⁶⁾ "A source of potent monocular depth information based on differences in relative motion between images of objects located at different distances from an observer." (Sekuler and Blake, 2002, p. $620)^{17}$)

"Relative movement of objects at different distances, a cue to depth" (Levine, 2000, p. $568)^{18}$).

"A depth cue. As an observer moves, nearby objects appear to move rapidly whereas far objects appear to move slowly" (Goldstein, 2002, p. 608)¹⁹⁾.

4. Summary of experimental findings relevant to the definitions

To comment on the relation between the textbook definitions and the experimental findings, it is useful to summarize the technique and the findings of Heine $(1905)^{9}$. By yoking the two stimulus movements to the body movement, he simulated two stationary stimuli. That is, he provided "identical incoming messages" or what Ames called "equivalent configurations" from different external physical arrangements (see Ittelson 1960)¹²⁾. With this understanding of relative retina image motion produced by a head movement, we now define the magnitude of parallax, which is the relative retinal motion per head movement. If we were to specify the extents in terms of visual angles (α and β or velocities of two retinal motions, the difference between the two divided by the extent or velocity of head movement is the parallax magnitude. If we compute $(\alpha - \beta$ when the head moves 6.2 cm, we get a measurement of what Rogers and Graham (1982)²⁰⁾ called "Equivalent Disparity" that is equal to the unit of retinal disparity for the identical depth at a given distance.

Rogers and Graham (1979)¹³⁾ simulated different stationary surfaces (square, sine,

triangular, or saw tooth) and observers saw stationary surfaces. Moreover, Ono, Rivest and Ono (1986)²¹⁾ also found that when magnitude of parallax is small the surface appeared stationary on the screen, but when it is large the whole fixed surface appeared to move. Therefore, apparent movement is not a necessary condition for depth perception generated by an observer's movement. As discussed above, yoking or "slaving" the stimulus movement to head movement provided insight into how a self-generated retinal motion leads to unambiguous depth perception and motion perception (By "unambiguous", we mean the direction of perception is stable, unlike that of the kinetic depth effect). When the parallax magnitude is relatively small, depth is seen without motion, and when it is large apparent motion is seen. This technique by itself, however, did not directly control the headmovement and therefore did not control the retinal motion. The instructions given in Heine $(1905)^{9}$ or Rogers and Grahams $(1979)^{13}$ were not known, but Ono et al. (1986)²¹⁾ instructed their observers to move their head from side to side at a "comfortable rate". Thus, the precise magnitude and velocity of head movement and corresponding retinal image motion in these studies were unknown.

To comment further on the textbook definitions, we summarize Ono and Ujike's (submitted)²⁾ method to study depth and motion perception produced by self-generated retinal motion. See Ono and Ujike $(1994)^{22}$ and Ujike and Ono $(2001)^{23}$, for the details of their technique. **Figure 2** illustrates the head movement profile and stimulus. The data were collected while the head was moving with a constant velocity, and the stimulus on the screen consisted of four horizontal bands of grating and each band moved in the opposite direction to its



Fig. 2. How we produced constant retinal velocity (A) and measured depth thresholds (B). The stimulus was presented only when the head (therefore the retinal image) was moving at a constant velocity. Figure 2B illustrates how we measured the depth threshold, but the experiments described in the text also measured the concomitant motion threshold and equal depth contours. For the concomitant motion threshold, observers adjusted the dial to find a border between depth with no motion and depth with motion. For equal depth contours, observers adjusted the dial to find the extent of depth that appeared equal to the depth produced by a standard stimulus. The figures are adapted from Ujike and Ono (2001)²³⁾.

adjacent band.

Figure 3 presents Ono and Ujike's (submitted)²⁾ results. Figure 3A shows the apparent depth magnitude for different parallax magnitudes obtained with relatively fast head movement (16 cm/s). It shows that the



Fig. 3. The magnitude of apparent depth and motion perception as a function of magnitude of parallax (3A) and an idealized illustration of five different zones (3B). There are zones in which an observer (a) perceived no depth and no motion, (b) perceived depth that is underestimated without perceived motion, (c) perceived correct depth without perceived motion, (d) perceived depth and concomitant motion, and (e) perceived motion only. The parallax magnitude is specified in terms of equivalent disparity. The figures are adapted from Ono and Ujike (submitted)².

magnitude of apparent depth increases as the parallax magnitude increases, until concomitant motion is seen. Then, with the perception of concomitant motion, the magnitude of apparent depth begins to decline as the parallax magnitude increases. When the parallax magnitude increases further, only concomitant motion is seen.

The following two findings suggest the apparent motion is not a cue to depth; (a) only depth (and no concomitant motion) is seen when parallax magnitude is small and (b) only concomitant motion (and no depth) is seen when the parallax magnitude is very large. This suggestion is parallel to the fact that, in a stereoscope, when retinal disparity is very large it is no longer effective as a cue to depth, and diplopia or rivalry rather than depth is seen. Moreover, there are further parallels with stereo depth as summarized by Tyler (1983)²⁴; the apparent depth magnitude increases first as the disparity increases (Panum's area), but with diplopia the apparent depth magnitude begins to decrease until it reaches the upper or terminal disparity threshold.

In Fig. 3B, idealized data represent depth threshold, concomitant motion threshold, equal depth "contours", and the estimated terminal depth threshold as a function of parallax magnitude for different head velocities. The equal depth contours measured how much equivalent disparity were required to produce the same depth with different head velocities. Several conclusions are derived from Fig. 3B. First, the depth threshold is dependent upon head velocity for slower head movements. For slow head movements (<≈13 cm/s) greater parallax magnitude is required to see the depth. Second, for the range of slower head movements, the motion threshold is the determinant of the depth threshold (see Ujike and Ono²³⁾, 2001 for the basis of this conclusion). The depth thresholds with negative slopes in the figure reflect the fact that when the head is moving faster it produces the same velocity of retinal image motion. Within this range, the assumption made by Graham et al. $(1948)^{10}$ and Zegers $(1948)^{11}$ that the motion threshold corresponds to the motion parallax threshold seems to be correct. Third, between the line for the depth threshold and that of the concomitant motion threshold, apparent depth is seen without motion. In between these two lines, location constancy holds. Fourth, for faster head movements ($>\approx13$ cm/sec), the depth threshold and the equal depth contours are determined by constant values of parallax magnitude. The zone in which the apparent depth agrees with the geometric prediction is relatively small (indicated by the grey shading). Moreover, the assumption made by Graham et al.¹⁰⁾ and Zegers¹¹⁾ does not apply for fast head movements. Fifth, above the concomitant motion threshold, both motion and apparent depth are seen, whereas above the terminal threshold for depth, only motion is seen.

Then how should one understand the apparent motion corresponding to the top two zones? One way is to consider the motion seen in these zones as the retinal image motion that the visual system cannot or does not "convert" into depth perception (Ono et al., 1986)²¹⁾. In the top zone in Fig. 3B, all the retinal image motion is allocated to apparent motion, whereas in the next zone, part of retinal image motion is allocated to motion and the rest is allocated to depth. Another finding is consistent with this idea. There is a trade off between depth and motion in the zone in which both depth and motion are seen (Ono and Steinbach, 1990; Sakurai and Ono, 2000)^{25,26)}. Presenting the same motion parallax display binocularly produced less depth and more motion than presenting it monocularly, since retinal disparity indicated a flat surface (Ichikawa and Saida, 1996; Rogers and Collett, 1989)^{27,28)} but more motion.

In the zone between the concomitant motion threshold and depth threshold perceptual stability exists. Location constancy holds for this zone but depth constancy is limited to the right hand (shaded area) of the zone. The zone of stability constitutes only a small portion of the visual field unlike stability of the whole visual field measured with the head stationary and the eyes moving. This limited stability and the top zone of only motion can be understood in terms of the visual system doing its best to deal with different demands. The stability and the seen depth with relatively high head velocity represent the perception of correct location (location constancy and depth constancy); the seen concomitant motion represents the correct perception of a change in headcentric direction. It is functional to see a stationary object as stationary near a fixation point; it is also functional to see the change in headcentric direction as well from a stimulus far away from the fixation. In this way of thinking, the middle zone in which depth and motion are seen is a mere transition zone between the two functional zones.

5. Discussion of the textbooks' definitions of motion parallax

None of the textbook definitions or the illustrations that accompany them suggests that the perception of apparent motion or depth is limited. Figure 3, however, indicates that there is a limit for both the perception of motion and that of depth at a given fixation point. This is not surprising in that retinal disparity, with which motion parallax is often compared, has a similar limit. Figure 3 also shows that there is a zone in which both motion and depth are seen. The depth and motion perceptions depend on the parallax magnitude, and both are an outcome of self-generated retinal image motion. This fact should be included in textbook discussions if not in a definition or illustration.

The definitions given state or imply that this apparent motion is a cue to depth. The experimental evidence and our daily experience clearly show that apparent concomitant motion is produced by self-generated retinal image motion. However, our examinations of the experimental findings suggest that a definition should not include the statement that apparent motion is the cue to depth. When the selfgenerated stimulus motion is present on a screen, apparent depth is not seen when the motion parallax magnitude is large, and there is as yet no experimental evidence that the apparent motion by itself is a cue to depth.

Most textbooks mention motion parallax as a depth cue, but it is the information from retinal image motion plus the head or eye movement information that provide a cue to depth. They do not make it explicit, however, that the direction of the head or eye movement information is used to disambiguate the direction of depth, and it is the magnitude of this information relative to retinal image motion that determines the magnitude of perceived depth for a given viewing distance.

All the illustrations and definitions surveyed by Ono and Wade (submitted)¹⁾ dealt with lateral head movement, lateral retinal image motion, or lateral apparent motion of stimuli. Probably, the popularity of the lateral movements among researchers and textbook writers is due to its possible comparison with binocular depth perception. Among researchers, for example, Rogers and Graham (1983)²⁹⁾ examined the same apparent surfaces produced by self-generated retinal image motion and those produced by retinal disparity, and Gonzalez, Steinbach, Ono, and Wolf (1989)³⁰⁾ examined depth perception combining head and no head movement conditions with monocular and binocular conditions. Among textbook writers, Wade and Swanston (2001)³¹⁾ used almost identical illustrations to discuss the processing of depth from self-generated retinal image motion and from retinal disparity.

The literature, however, shows that an upand-down or a forward-and-backward head movement when yoked to an appropriate retinal image motion is also effective in producing depth (Steinbach, Ono and Wolf, 1991; Sakurai and Ono, 2000; Yajima, Ujike and Uchikawa, 1998)^{26,32,33)}. Moreover, the retinal image motion is not always in the direction of the head movement; a forward-and-backward head movement or lateral head movement with respect to a slanted surface produces retinal image motion of expansion and contraction and leads to reliable depth perception (Rogers and Graham, 1983; Sakurai and Ono, 2000)^{26,29)}. Therefore, it may not be prudent to restrict the definition to a lateral head movement, particularly given the frequently cited statement by Helmholtz (2000)³⁴⁾ that considers forward movement, but using an illustration with a lateral head movement may be appropriate as an example of self-generated change in retinal images leading to depth perception. Our suggestion is to make the definition general enough to include a wider range of head movements and to include expanding and contracting retinal stimuli.

The literature also shows that viewing distance is an important variable, because of the inverse square law (e.g., Ono et al., 1986)²¹⁾. This point is made clear only in the figure caption of Wade and Swanston $(2001)^{31}$, but whether all textbook definitions should include the geometry is, again, debatable. A discussion of this geometry in the accompanying text is worth considering. Perhaps, a useful guideline is that when the geometry is discussed for binocular vision it should also be discussed for motion parallax, or vice versa.

Finally, some definitions (e.g., Sekuler and Blake, 2002; Levine, $2000)^{17,18}$ do not exclude other motion phenomena such as the kinetic

depth effect, the stereokinetic effect, or structure from motion. These three effects, however, are based on retinal image motion created by moving stimuli, whereas what we consider in this paper is retinal image motion created by stationary stimuli viewed by a moving observer. Moreover, for these three effects, motion perception is necessary and the direction of depth is ambiguous. Since most textbooks define and discuss these effects as well, we suggest that the definition of motion parallax be restricted to self-generated retinal image motion (cf. Howard and Rogers, 2002)¹⁵⁾. The information from self-generated retinal image motion is thought to combine with the extraretinal information from head-movement and eve movement to produce depth perception. Presumably, when none of the self-generated retinal image motion is allocated to depth, the extra-retinal signal is ignored. Although the retinal image motion for the three effects can be geometrically related to parallax, the source of this information is defined in terms of retinal image movement alone. Furthermore, from a pedagogical point, there is an advantage in a definition that is specific enough to distinguish it from other phenomena. Our opening historical survey also argues for restricting the definition of motion parallax to perceived depth and concomitant motion produced by the selfgenerated retinal image motion.

6. Recommendation

On the basis of the discussions above, several recommendations regarding the definition of motion parallax can be made. Some are positive, indicating what should be included; others are negative, suggesting what should not be implied. On the positive side, we recommend that the definition (a) state that the self-generated retinal image motion is the proximal stimulus, (b) include both perceived depth and perceived concomitant motion, and (c) state that the perceptions of motion or depth depend on viewing distance, parallax magnitude, and head velocity. On the negative side, we suggest that the definition should not (a) state or imply that apparent motion is a cue to depth, and (b) be restricted to lateral head movement.

Accordingly, we suggest the following definition that is simple and meets the recommendations. It also does not detail the experimental complexities described above. Motion parallax refers to retinal image motion generated by head movements relative to stationary objects at different distances; the objects will be seen in depth and/or will appear to move, depending on fixation distance and the velocities of retinal image and head movements.

7. Authors' notes

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